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CONCERNING NUMERICAL SIMULATION OF THE FAILURE OF A MAGNETIC-FLUID SEAL WITH A ROTARY OUTER PROFILED CYLINDER

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The behavior of a circular magnetic fluid seal in the narrow gap between an immovable shaft and a rotary hyperbolic concentrator of the magnetic fluid flow surrounding it is studied. The range of the dimensionless rotation parameter within which the seal is stable is determined.

1. Introduction. In magnetic-fluid seals (MFS) of rotary shafts, the magnetic fluid (MF) is held by the high-gradient magnetic field created by a circular concentrator of the magnetic flux surrounding the shaft. Magnetic forces ensure equilibrium of the MF volume owing to the external pressure drop and centrifugal forces. A poorly studied important factor is deformation of the free MF surface at high rotation velocities of the shaft. Usually the surface shape is either prescribed [1, 2] or determined in an approximation of the prescribed velocity distribution ignoring capillary forces [3, 4]. Calculations, however, show that even a comparatively low shaft rotation velocity can cause substantial deformation of the free surface. As a result, the velocity in the MF volume is redistributed thus exerting, in turn, a pronounced influence on surface formation.

A solution of the problem in the absence of an external pressure drop was begun earlier [5, 6]. There the case is considered in which a magnetic gap is formed by the smooth surface of the rotary shaft and an immovable profiled surface, i.e., a concentrator of the magnetic flux. Simulation of the free MFS surface is accomplished by solving numerically a system of integro-differential equations whose complicated form makes it difficult to obtain an analytical solution even in simple situations.

Of particular importance is determination of the critical rotation velocities at which the fluid is thrown from a magnetic gap by centrifugal forces. In [5, 6], the onset of an MFS crisis is indicated by instability in the iteration process for solution of the steady-state problem of equilibrium forms of a free surface. In [7, 8] this method was extended to investigation of drop equilibrium in gravitational, uniform magnetic fields and in a potential field of centrifugal forces and tested in the cases that permit an analytical solution. One of the goals of the present work was to substantiate the reliability of such simulation of the free MFS surface instability developed due to the action of nonpotential centrifugal forces. In the model, the smooth inner cylinder was immovable, while the profiled outer cylinder rotated. In this case, a theoretical analysis of the free surface stability and analytical estimates for critical parameters are possible.

2. General Equations. The mathematical model is formulated rather fully in [5, 6]. Here we discuss in brief its main statements. The model is based on a system of hydrodynamic equations of an incompressible, isotropic, linear-viscous fluid with constant transfer coefficients which is supplemented with the force of interaction with a magnetic field in an approximation of equilibrium magnetization [9]. In the case of a laminar flow and in the absence of eccentricity between the concentrator and the shaft, magnetic and hydrodynamic forces possess axial symmetry. However, since the relative width of the magnetic gap is small, a two-dimensional approximation is reasonable when the equations of motion are written in the local Cartesian coordinate system and the rotational character of motion is allowed for by conservation of the centrifugal force.

Let the coordinate origin be on the shaft surface (assumed to be two-dimensional) with x the radial, z the axial, and y the azimuthal directions (see Fig. 1).

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A field of centrifugal forces in the fluid has no potential in the general case, and the eddy component of this field generates a secondary flow. However, its intensity is low as compared to the main azimuthal flow. Ignoring the influence of the secondary flow, the distribution of the azimuthal velocity, as follows from the Navier-Stokes equation for straight-line stationary motion, satisfies the Laplace equation

$$\nabla^2 v = 0 \tag{1}$$

and the boundary conditions

$$v\big|_{l^{(p)}} = 0\,, \quad v\big|_{l^{(c)}} = v_0\,, \quad v_{,n}\big|_{l^{(s)}} = 0\,, \tag{2}$$

where $l^{(p)}$, $l^{(c)}$, and $l^{(s)}$ are the axial cross-sections of a plane surface, concentrator, and free surface, respectively. The first two conditions from (2) are those of fluid adhesion to solid walls, while the third condition indicates the absence of shear stresses on the free surface.

In accordance with the potential theory [10] the solution of problem (1) and (2) on the closed contour l confining the axial cross section of the magnetic-fluid seal satisfies the following integral equation:

$$\pi v = \oint_l (u_{,n}\bar{v} - u\bar{v}_{,n})\, d\bar{l}\,, \tag{3}$$

where $u = \ln \xi$; ξ is the distance between the source point $(\bar{x}, \bar{z})$ and the observation point (x, z) of the contour l; n is the external normal at the source point; $\bar{v}$ and v are the velocities at the source point and at the point of observation; $d\bar{l} = (d\bar{x}^2 + d\bar{z}^2)^{1/2}$.

The form of the free MF surface will be described by parametric equations $x(s)$, $z(s)$, where s is the length of the contour arc $l^{(s)}$ read from the shaft surface.

From the condition of pressure constancy due to volumetric magnetic and centrifugal forces as well as to magnetic and capillary pressure jumps it follows that the equation of the free surface has the form

$$\sigma\kappa = C + \mu_0 \left(\int_0^H MdH + M_n^2/2 \right) + \frac{\rho}{r_0} \int_0^x v^2 dx\,, \tag{4}$$

where $\kappa = -z''/x' = x''/z'$, the prime indicates differentiation with respect to s; C is an arbitrary constant determined from additional conditions.

3. Closed Mathematical Model. Let us introduce dimensionless variables, choosing as characteristic scales the minimum gap width l_0 for distances and the concentrator velocity v_0 for velocities.

Now we write Eq. (3) in terms of dimensionless variables with allowance for conditions (2) for observation points on the concentrator surface $l^{(c)}$, the plane shaft surface $l^{(p)}$, and the free surface $l^{(s)}$, respectively:

$$\begin{gathered}
\int_{l^{(p)} + l^{(c)}} u\bar{v}_{,n}\, d\bar{l} - \int_{l^{(s)}} u_{,n}\bar{v} d\bar{l} = \int_{l^{(c)}} u_{,n} d\bar{l} - \pi\,; \\
\int_{l^{(p)} + l^{(c)}} u\bar{v}_{,n}\, d\bar{l} - \int_{l^{(s)}} u_{,n}\bar{v} d\bar{l} = \int_{l^{(c)}} u_{,n} d\bar{l}\,; \\
\int_{l^{(p)} + l^{(c)}} u\bar{v}_{,n} d\bar{l} - \int_{l^{(s)}} u_{,n}\bar{v} d\bar{l} + \pi v = \int_{l^{(c)}} u_{,n} d\bar{l}\,.
\end{gathered} \tag{5}$$

We consider the magnetic gap formed by a plane surface and a hyperbolic concentrator of the magnetic flow. In this case, a dimensionless equation of the contour $l^{(c)}$ is of the form

$$x^2/\cos^2\beta - z^2/\sin^2\beta = 1/\cos^2\beta$$

(see Fig. 1). Such a form of the concentrator allows an analytical expression to be obtained for an external magnetic field that reflects the main features of the field distribution in the magnetic gap of a real MFS.

We assume that the shaft and the concentrator are far from magnetic saturation and their magnetic permeability is $\mu >> 1$, which allows us to consider the magnetic potential at each of the gap boundaries to be constant. Then the field distribution in the gap is described by the formula

$$\mathbf{H} = H_c \sin\beta\,(\cos\delta\,\mathbf{e}_x - \sin\delta\,\mathbf{e}_z)/g\,, \tag{6}$$

where $\mathbf{e}_x$ and $\mathbf{e}_z$ are unit vectors of the coordinate axes;

$$\cos\delta = \frac{\zeta\,(1-\tau^2)^{1/2}}{g}\,, \quad g = (\zeta^2 - \tau^2)^{1/2}\,,$$

$$\zeta = 0.5\cos\beta\,(r^+ + r^-)\,, \quad \tau = 0.5\cos\beta\,(r^+ - r^-)\,, \quad r^\pm = \left((x \pm 1/\cos\beta)^2 + z^2\right)^{1/2}.$$

From (4) with allowance for (6) for the case of magnetic saturation ($M = M_s = \text{const}$) we obtain a system of dimensionless differential equations

$$\mathrm{Bo}_m^{-1} z'' = -x'(f + C)\,, \quad \mathrm{Bo}_m^{-1} x'' = z'(f + C)\,, \tag{7}$$

where

$$f = F\int_0^s v^2 x' ds + \frac{\sin\beta}{g}\left[1 + \mathrm{Xi}\,\frac{\sin\beta}{g}\,(x'\sin\delta - z'\cos\delta)\right].$$

On the shaft and concentrator surfaces the kinematic conditions

$$x_0 = 0\,, \quad z_1 = (x_1^2 - 1)^{1/2}\tan\beta \tag{8}$$

and conditions of constancy of the wetting angle

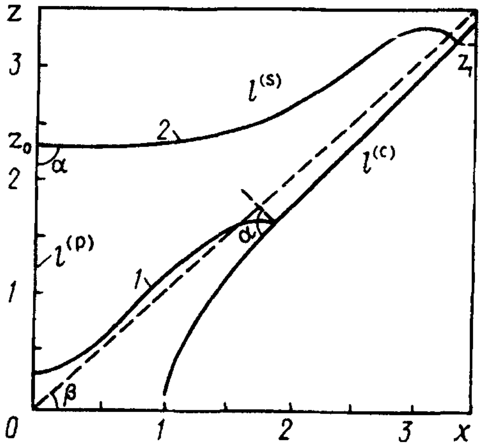
$$z_0' = -\cos\alpha\,, \quad x_0' = \sin\alpha\,, \quad z_1' = \cos(\delta_1 + \alpha)\,, \quad x_1' = \sin(\delta_1 + \alpha)\,; \tag{9}$$

are fulfilled, where subscripts 0 and 1 stand for the quantities at the initial ($s = 0$) and end ($s = S$) points of the free surface; $S = l_s/l_0$; l_s is the length of the contour $l^{(s)}$. Integrating the first of equations (7) with allowance for the boundary conditions, we arrive at an expression for the constant C:

$$C = -\frac{1}{x_1}\left[\mathrm{Bo}_m^{-1}\,(\cos(\delta_1 + \alpha) + \cos\alpha) + \int_0^S f x' ds\right]. \tag{10}$$

A magnetic fluid seal has two free surfaces. We shall consider the case when the pressure drop of the external medium on these surfaces is equal to zero. In this situation the axis x is an axis of bilateral symmetry, so that only one free boundary, in fact, is to be calculated and the condition of volume constancy can be written in the form:

$$U = 2\int_0^S z x' ds - x_1 z_1 + \frac{1}{2}\tan\beta\,\ln\frac{x_1\tan\beta + z_1}{x_1\tan\beta - z_1}\,. \tag{11}$$



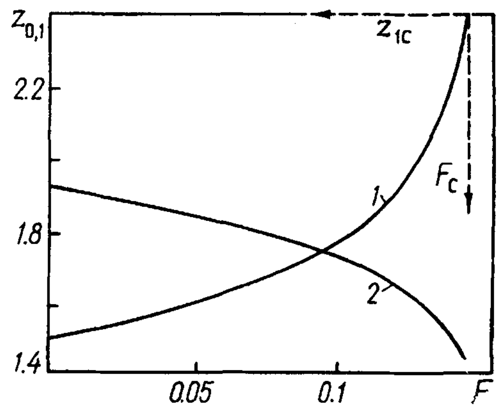


Fig. 1. Geometry of the problem. Free surface configuration at 1) $U = U_* = 2$, $F = F_* = 0.246$; 2) $U = 9$; $F = F_c(U) = 0.082$.

Fig. 2. Dependences of the z-coordinate of the points of contact of the free surface with the profiled (1) and plane (2) gap walls on the rotation parameter at $U = 5$.

The system of equations (5), (7)-(11) represents a closed mathematical model of a problem within the framework of which the free surface is determined by six parameters, namely, the dimensionless volume U, the rotation parameter F, which expresents the ratio of centrifugal to magnetic forces, the magnetic Bond number Bo_m, the characteristic magnetic susceptibility Xi, the wetting angle α, and the angular halfwidth β of the concentrator.

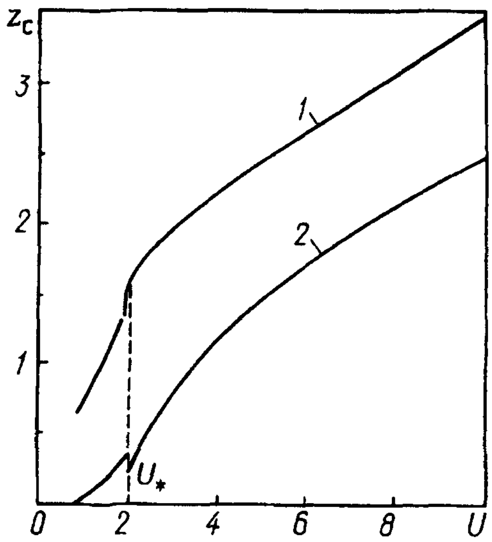
The problem (5), (7)-(11) was solved numerically by the method of successive approximations [5, 6]. In each iteration run system of integral equations (5) was at first solved by the method of boundary elements [11], and the velocity on the free surface found in the previous iteration was determined. The position of the surface was then determined more precisely with allowance for the new velocity distribution by solving problem (7)-(11) using an algorithm similar to [7]. Computational instability was successfully eliminated with the help of relaxation parameters following the procedure described in [12].

4. Analysis of Numerical Results. The data reported below were obtained at constant values of $\beta = \pi/4$, $Xi = 2\pi \cdot 10^{-2}$, $Bo_m = 200$, $\alpha = \pi/2$ typical for MFS conditions. The rotation parameter F and "volume" U were varied.

Figure 1 shows the results of a typical calculation of the free-surface configuration. It is convenient to evaluate the influence of external parameters on the surface configuration from the coordinates z_0 and z_1, the points of its contact with the plane and profiled, respectively, walls of the gap. As is seen from Fig. 2, these coordinates depend on the rotation parameter F rather greatly, even at its compartatively small values. It is also worth noting that the derivatives dz_0/dF and dz_1/dF increase in absolute value with F.

At some critical values of $F = F_c(U)$, which are dependent on the volume, the algorithm for solution of the steady-state problem acquires computational instability, which is interpreted as a manifestation of the physical instability of the surface. Calculations have shown that the dependence $F_c(U)$ is determined by the competition of two instability mechanisms, one of which dominates at $U < U_*$, and the other at – at $U > U_*$, where U_* is the volume corresponding to the maximum rotation parameter $F_* = F_x(U_*)$ prior to which the magnetic-fluid seal can be in equilibrium.

In the region $U > U_*$ at $F \to F_c$ the values of z_0 and dz_0/dF remain finite, while $|dz_1/dF| \to \infty$. This allows us to conclude that in the case of large volumes the instability is caused by displacement of the contact point z_1 over the concentrator surface. Dependences of the critical $z_{1c}(U)$ values and corresponding $z_0(U)$ values are represented in Fig. 3, while those of the critical rotation parameters $F_c(U)$ are shown in Fig. 4. The physical mechanism of the instability at $U > U_*$ is as follows. As z_1 increases, the centrifugal ejecting force at this point grows, while the gradient of the magnetic force and, consequently, the magnetic force holding the fluid in the gap decrease. At



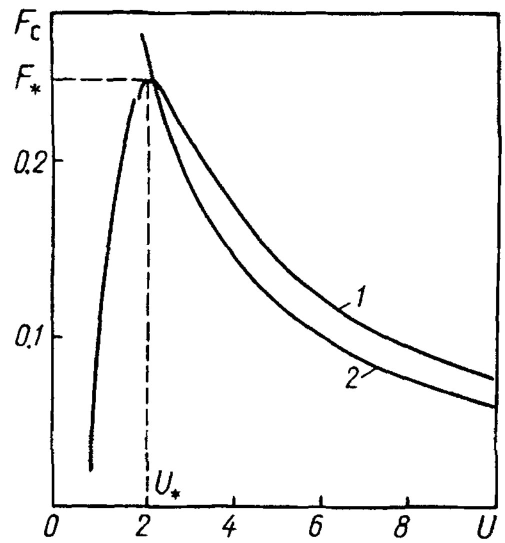


Fig. 3. Dependences of the critical values of the z-coordinate of the points of contact on the fluid volume: 1) z_{1c}; 2) z_{0c}.

Fig. 4. Calculated (1) and analytical (2) dependences of the critical values of the rotation parameter on the fluid volume.

the critical point z_{1c} the components of these forces tangential to the concentrator surface are counterbalanced. The condition of this equilibrium is as follows:

$$\mu_0 M_s \frac{dH}{dl^{(c)}} + \rho \frac{v_0^2}{r_0} \sin \delta_1 = 0 .$$

Hence, with allowance for (6) we find

$$F_c = \frac{\cos \beta \, (z_{1c}^2 + \tan^2 \beta)^{1/2}}{(z_{1c}^2 \operatorname{ctan}^2 \beta + \sin^2 \beta)^{3/2}} . \tag{12}$$

With a further increase the rotation parameter, the fluid is partially ejected from the gap. As a result of decreasing volume U, the critical F_c value increases (provided that the new volume is also in the interval $U > U_*$).

The $F_c(U)$ curve constructed with use of analytical formula (12) and numerical values of z_{1c} is shown in Fig. 4 along with the same dependence obtained numerically. For large volumes these dependences are seen to agree. The overestimation of the numerical values is attributable to allowance for surface tension forces, which hinder fluid ejection.

On passing to the region of "small" volumes $U < U_*$, the critical rotation parameter F_c begins to decrease with decreasing U. The qualitative change in the dependence is due to a change in the instability mechanism. At $U < U_*$ the point z_0 of contact of the free surface with the shaft is close to the singular point $z = 0$ at which the tangential component of the volume forces changes sign. The physical mechanism of the instability realized upon reaching the singular point consists in separation of the magnetic-fluid seal from the shaft surface and formation of the fluid layer forced against the concentrator surface by centrifugal forces. The considerable discrepancy between the analytical and numerical results in the region of small volumes in Fig. 4 is explained by the fact that analytical dependence (12) does not allow for this mechanism.

It is interesting that the two branches of the plots of the critical parameters (see Figs. 3, 4) corresponding to instability different mechanisms are joined in the presence of appreciable discontinuities. The latter are reproduced both in the build-up of fluid volume in the course of numerical solution and in its decrease as well.

Analysis of numerical results shows that obtaining critical F values with respect to computational instability gives quite an adequate representation of the onset of physical failure of MFS with increasing rotation velocity of the shaft. The important result, we believe, is determination of the limiting value of the critical rotation parameter $F_* = 0.246$ and the corresponding dimensionless volume $U_* = 2$.

From the definition of the dimensionless parameters F and U we have the following expressions for the limiting rotation velocities of the concentrator and the corresponding optimum volumes of the fluid

$$v_{max} = \sqrt{F_* \mu_0 M_s H_c r_0 / (\rho l_0)}\ , \quad V_* = U_* 2\pi r_0 l_0^2 .$$

In the general case, the parameters F_* and U_* depend on the wetting angle α and the angular halfwidth β of the concentrator. The values obtained at $\alpha = \pi/2$ and $\beta = \pi/4$ can be chosen as reference quantities for determination of more refined dependences $F_*(\alpha, \beta)$, $U_*(\alpha, \beta)$.

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NOTATION

x, z, Cartesian coordinates; κ, curvature of the free surface; H, magnetic-field intensity; M, fluid magnetization; v, azimuthal velocity of fluid motion; v_0, rotation velocity of outer cylinder; μ_0, magnetic constant; M_s, saturation magnetization of fluid; σ, surface tension coefficient; ρ, fluid density; V, fluid volume; α, wetting angle; r_0, radius of inner cylinder; l_0, minimum gap width; β, angular halfwidth between the asymptotes of the concentrator hyperbola; $\mathrm{Bo_m} = \mu_0 M_s H_c l_0/\sigma$, magnetic Bond number; $U = V/(2\pi r_0 l_0^2)$; $F = \rho v_0^2 l_0/(\mu_0 M_s H_s r_0)$; $\mathrm{Xi} = M_s/H_c$; $H_c = \varphi_0 \operatorname{ctan}\beta/[l_0(\pi/2 - \beta)]$, intensity of the magnetic field at the concentrator top; φ_0, difference in magnetic potentials on the profiled and plane walls.

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